Digital Communications II

Third Year, 2ed Semester

Lecture No. 7

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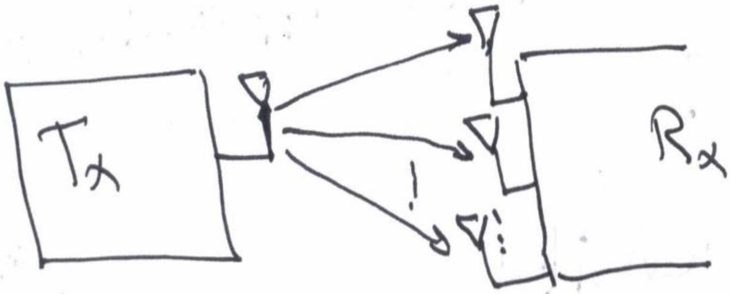
Model of Multiple Antenna Systems

System Model of Multiple Antenna System



L^{th} order diversity — 1 Transmit antenna, L Receive antennas

$$y = hx + n$$ --- Wireless

L such link:

$$y_1 = h_1 x + n_1$$ Model between Tx Antenna and Rx Antenna 1

$$y_2 = h_2 x + n_2$$

$\vdots$

$$y_L = h_L x + n_L$$

h_1 is fading coefficient of Link 1

h_2 is fading coefficient of Link 2

$\vdots$

h_L is the fading coefficient of link L

System Model:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix} \leftarrow \text{nois vector}$$

$$\bar{y} \qquad \bar{h} \qquad \bar{n}$$

$$\boxed{\underset{L\times 1}{\bar{y}} = \underset{L\times 1}{\bar{h}}\, x + \underset{L\times 1}{\bar{n}}}$$ vector notation

* Combine these 2 ~~syst~~ symbels.

combining weights

$$\tilde{y} = w_1 y_1 + w_2 y_2$$

$$= [w_1 \;\; w_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \bar{w}^T \bar{y}$$

$$\boxed{\bar{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}$$

$$\boxed{\bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}}$$

$$\tilde{y} = \bar{w}^T (\bar{h} x + \bar{n})$$

$$= \underbrace{\bar{w}^T h x}_{\text{signal component}} + \underbrace{\bar{w}^T \bar{n}}_{\text{noise compoment}}$$

$$\boxed{\|\bar{w}\|^2 = w_1^2 + w_2^2}$$

$$\boxed{SNR = \frac{|\bar{w}^T \bar{h}|^2 P}{E\{|\bar{w}^T \bar{n}|^2\}}} = \boxed{\frac{|\bar{w}^T \bar{h}|^2 P}{\sigma^2 \|\bar{w}\|^2}} \; SNR$$

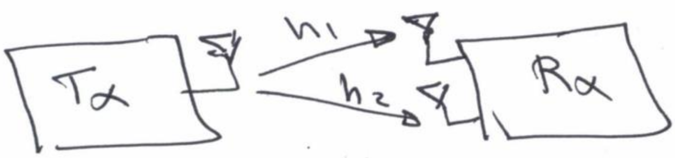
$$\bar{w}^T \bar{h} = [w_1 \;\; w_2] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = w_1 h_1 + w_2 h_2 = \bar{w} \cdot \bar{h}$$

$$\bar{w} = \frac{\bar{h}}{\|\bar{h}\|} = \frac{1}{\sqrt{|h_1|^2 + |h_2|^2}} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \leftarrow \text{Maximal Ratio Combining}$$

$$\boxed{\text{MRC}}$$

(13)

Example:



$$h_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \qquad y_1 = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j\right) x + n_1$$

$$h_2 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \qquad y_2 \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j\right) x + n_2$$

$$E\{|n_1|^2\} = E\{|n_2|^2\} = \sigma^2 = \frac{1}{2}$$

$$10 \log_{10} \frac{1}{2} = -3\,dB$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\bar{y} = \bar{h} x + \bar{n}$$

The optimal MRC vector $= \dfrac{\bar{h}}{\|\bar{h}\|}$

$$\|\bar{h}\| = \sqrt{|h_1|^2 + |h_2|^2} = \sqrt{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = \sqrt{2}$$

$$\bar{w} = \frac{\bar{h}}{\|\bar{h}\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix} \quad \leftarrow \text{Maximal Ratio combining vector}$$

$$\boxed{SNR = \frac{|\bar{w}^H \bar{h}|^2 P}{\sigma^2 \|\bar{w}\|^2}}$$

[illegible]

$$\bar{w}^H \bar{h} = \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \quad \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j\right] \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix}$$

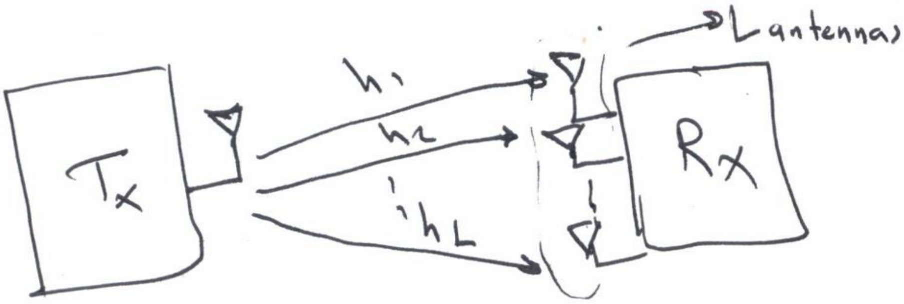
$$= \frac{1}{\sqrt{2}} \left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right]$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2} \qquad \therefore |\bar{w}^H \bar{h}|^2 = (\sqrt{2})^2 = 2$$

$$\|\bar{w}\|^2 = |w_1|^2 + |w_2|^2 = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right) = 1$$

$$SNR = \frac{P |\bar{w}^H \bar{h}|^2}{\sigma^2 \|\bar{w}\|^2} = \frac{P \cdot 2}{\frac{1}{2} \cdot 1} = \boxed{4P} \text{ for MRC}$$

Generalize MRC to L Antennas



channel coefficients = $h_1, h_2, \ldots h_L$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

$$\bar{y} = \bar{h} x + \bar{n} \quad \rightarrow \text{nois power} = \sigma^2$$

$$E\{|n_i|^2\} = \sigma^2 \qquad E\{n_i n_j\} = 0 \quad i \neq j$$

nois on any pair of antenna is uncorrelated

$$\tilde{y} = w_1^* y_1 + w_2^* y_2 + \cdots w_L^* y_L$$

$$= \bar{w}^H \bar{y} = \bar{w}^H(\bar{h} x + \bar{n}) = \underbrace{\bar{w}^H \bar{h} x}_{\text{signal}} + \underbrace{\bar{w}^H \bar{n}}_{\text{noise}}$$

For maximum SNR, choose MRC

$$\bar{w} = \frac{\bar{h}}{\|\bar{h}\|} = \frac{1}{\|\bar{h}\|} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} \quad \text{and } \|\bar{h}\| = \sqrt{|h_1|^2 + |h_2|^2 + \cdots |h_L|^2}$$

$$\boxed{SNR = \frac{\|\bar{h}\|^2 P}{\sigma^2}}$$

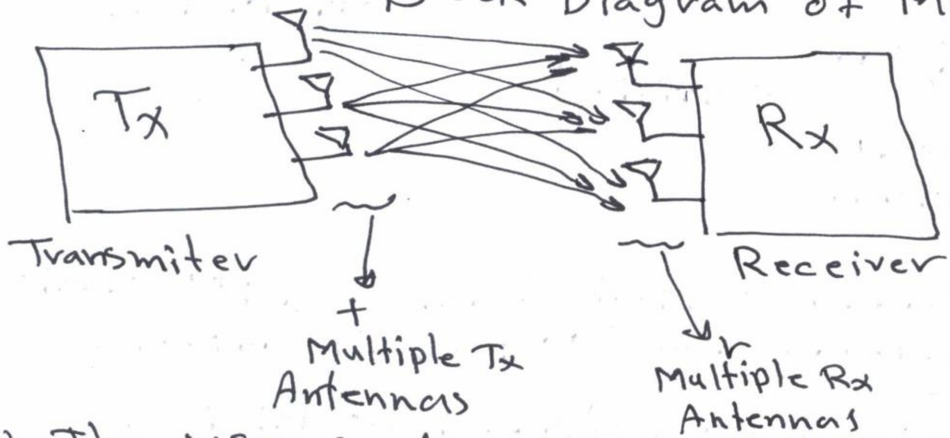
Multiple Input Multiple Output Systems MIMO

MIMO ⟹ Increase the Data Rates

Multiple Input ⟹ Multiple Transmit Antennas

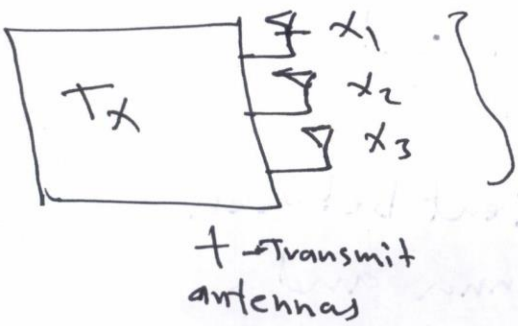
Multiple output ⟹ Multiple Receive Antennas

Block Diagram of MIMO



So the MIMO System is a collection of a large number of fading channels

Model For MIMO



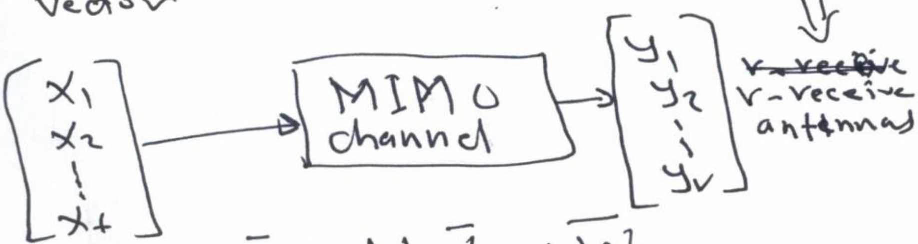
$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} \Rightarrow t\text{-dimensional vector}$$

⇓ Transmit vector

$$\bar{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix}$$

r-dimensional vector

⇓



$$\bar{y} = H\bar{x} + \bar{w}$$

$\bar{y}$: $r \times 1$; H: $r \times t$ Matrix; $\bar{x}$: $t \times 1$; $\bar{w}$: $r \times 1$

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \\ h_{21} & h_{22} & \cdots & h_{2t} \\ \vdots & & & \\ h_{r1} & h_{r2} & \cdots & h_{rt} \end{bmatrix}$$

r-row

t-columns

H-Matrix

$h_{ij} \longrightarrow i,j^{th}$ coefficient

$i^{th} \Rightarrow$ row

$j^{th} \Rightarrow$ column

So $h_{ij} \Rightarrow$ channel coefficient between i^{th} receive antenna and j^{th} transmit antenna

and the Total number of channel coefficient

$$= r \times t$$

* $h_{32} \Rightarrow$ coefficient between 3rd receive antenna and 2nd trasmit antenna

* $h_{23} \Rightarrow$ channel coefficient between 2nd receive antenna and 3rd transmit antenna

In general $r \times t$ system

$$y_l = h_{l1} x_1 + h_{l2} x_2 + \cdots h_{lt} x_t + w_l$$

⇓

Symbol receive ~~receiv~~ on 1^{th} antenna

Special Cases: $t = 1$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_r \end{bmatrix} x + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_r \end{bmatrix} \Leftarrow \text{SIMO}$$

Multiple receive antennas

SIMO: Single Input Multiple output

$r = 1$

$$y_1 = [h_1 \; h_2 \; \cdots \; h_t] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} + w_1 \Leftarrow \text{MISO}$$

channel vector

$$y_1 = \bar{h}^H \bar{x} + w_1 \quad , \quad \bar{h} = \begin{bmatrix} h_1^* \\ h_2^* \\ \vdots \\ h_t^* \end{bmatrix} \quad \bar{h}^H = [h_1 \; h_2 \cdots h_t]$$

$r = 1 \, , \, t = 1$

$$y = hx + w \Leftarrow \text{SISO}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_t \end{bmatrix}$$ r – Dimensional noise vector

w_i is Gaussian

→ mean $= 0$

→ verTance σ^2

Example of MIMO Systems

Example:

$r = 3$ Receive antenna
$t = 2$ Transmit antenna

So 3×2 MIMO System

Transmit vector $= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ (2-Dimensional)

- x_1 → Transmitted on 1st antenna
- x_2 → Transmitted on 2nd antenna

Receive vector $= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ (3-Dimensional)

- y_1 → Received on 1st antenna
- y_2 → " " 2nd "
- y_3 → " " 3rd "

$H = 3 \times 2$ Matrix
$\quad r \quad t$ 6-coefficient

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

3-Dim $\bar{y}$ 3×2 matrix H 2-Dim $\bar{x}$ 3-Dim $\bar{w}$

$$y_1 = h_{11} x_1 + h_{12} x_2 + w_1$$
$$y_2 = h_{21} x_1 + h_{22} x_2 + w_2$$
$$y_3 = h_{31} x_1 + h_{32} x_2 + w_3$$